Statistics of Fields in Nonlinear Wave Chaotic Systems

The Random Coupling Model Approach

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Abstract—The Random Coupling Model (RCM) has been shown to successfully predict the statistical properties of linear wave chaotic cavities in the highly over-moded regime [1]. It is of interest to extend the RCM to strongly nonlinear systems.

Nonlinearity manifests as harmonic generation, amplitude dependent responses, etc. We have recently studied the statistics of harmonics by adding an active frequency multiplier to the ¼-bowtie microwave billiard [2]. To create amplitude dependent responses, we have introduced different sources of nonlinearity into the billiards. Observing nonlinearity usually requires that we be in the high amplitude regime, hence a high power vector network analyzer (VNA) is implemented to measure the S-parameters up to ~+40 dBm.

In this talk, we show the results of the nonlinear S-parameters in two nonlinear systems. One system is a diode-loaded ¼-bowtie microwave cavity where the diode acts as point nonlinearity in a wave chaotic system. By attaching a diode to the excitation port, we observed the statistics of the impedance change substantially with the excitation power as shown in Fig 1. We also find that the short orbits [3] between the port and a nearby wall are strongly modified. Other un-expected changes are also observed. We found that many of these changes are due to the fact that the admittance of the diode changes with the excitation power. The nonlinear diode competes with the cavity admittance, substantially altering the response of the system. By implementing the lossy port model extension of the RCM [4], the results are well explained by the changing radiation efficiency of the diode-loaded port. This configuration may have potential application in protecting electronic circuits from high power electromagnetic interference (EMI).

Another source of nonlinearity in microwave billiards is the nonlinear surface impedance $Z = R + iX$ of a material on the interior surface of the cavity. In particular we consider nonlinear superconducting materials coating the walls of a cavity. A cut-circle quasi-2D microwave cavity, which is made of Pb-plated copper, can present nonlinear boundary conditions at temperatures $T < T_C$, where $T_C = 7.2$ K. We have previously characterized the linear response properties of this extremely low loss system ($\alpha = 0.02$) at 6.6 K using an *in-situ* broadband cryogenic calibration technique [5]. Now by putting the cavity in a dilution refrigerator, the base temperature reaches 500 mK, allowing us to enter a strongly nonlinear regime. The power dependent S-parameters show that the cavity has a mainly resistive nonlinear response. The quality factor, Q, of many of the modes decreases as input power increases.

Applying RCM, we find that the impedance statistics also change with power. The results of this analysis may give insight into the statistical properties of generic nonlinear systems.

Furthermore, as a complement to the nonlinear resistive superconducting billiard, we are also proposing an experiment on a TiN-coated Si wafer cut-circle billiard, where the TiN superconducting films are reported to have a dominant nonlinear reactive response. We will report on the first measurements of this novel nonlinear system.

Keywords: electromagnetic interference, statistical electromagnetics, Random Coupling Model, wave chaos, nonlinear resonators

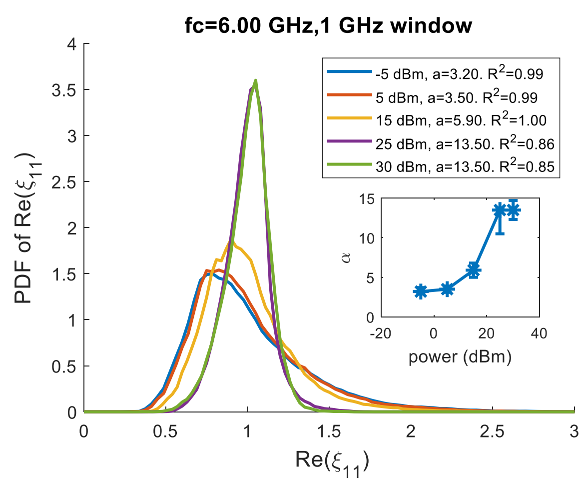

Figure 1.Statistics of RCM-normalized impedance ξ in a diode loaded bowtie for different excitation powers.

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