Quantum Graph Treatment of Electromagnetic Topology

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Abstract**—** The problem of how electromagnetic waves couple and propagate through complex structures such as buildings, aircraft, ships, etc. is very challenging. A number of approaches have treated the problem in terms of scalar energy flow through interconnected compartments [1]. These methods ignore wave effects (e.g. interference), and therefore provide only a mean-field description of energy distribution. We want to extend these approaches by including wave propagation effects and interference to understand deviations from the mean.

Quantum graphs provide a setting to formulate a more complete model of energy propagation in complex structures. Such graphs are a natural outcome of the Electromagnetic Topology approach [2]. There is a hope that the statistical properties of wave and energy distribution on these graphs can be understood using an approach based on the Random Coupling Model.

A graph or network is a set of elements which are connected in a certain topology. Graphs have applications in many different branches of engineering, science, sociology and biology [3]. A quantum graph, introduced by Pauling in the 1930s, is a linear network structure of vertices connected by bonds with a differential or pseudo-differential operator acting on functions defined on the bonds. Quantum graphs have been used to model many phenomena, such as acoustic and electromagnetic waveguide networks, quantum Hall systems and mesoscopic quantum systems [3].

Researchers have studied quantum graphs experimentally and numerically [4]. Quantum graphs have been realized as microwave networks with different topologies such as tetrahedral, irregular hexagon fully connected networks, and fully connected five vertex networks [4]. The impedance statistics of networks of complex enclosures [5] have been studied, and results from both numerical calculation and experimental measurement show good agreement with theory.

We employ an experimental setup consisting of a microwave coaxial cable network, which is used to simulate quantum graphs. The networks, which are large compared to the wavelength, are constructed from coaxial cables connected by T junctions. The distributions of impedance statistics are obtained from experiments on an ensemble of tetrahedral networks. The Random Coupling Model (RCM) is applied in an attempt to uncover the universal statistical properties of the experimental data obtained from this system. Deviations from RCM predictions have been observed in that the statistics of diagonal and off-diagonal impedance elements are different. It is argued that because of the small finite-size quantum graphs utilized here there will be non-universal results [6].

We have extended these simple microwave graphs by replacing the T junctions with quasi-two-dimensional microwave billiards that display wave chaotic properties. This models realistic scenarios in which over-moded enclosures are interconnected through multiple ports, including cables, apertures and waveguides (ducts). We will relate the results of these experiments to the general question of how to estimate induced voltage statistics on electronics deep inside a network of complex enclosures.

Keywords: electromagnetic interference, statistical electromagnetics, electromagnetic topology, Random Coupling Model, wave chaos

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